α Using Integer Tiles: Multiplication & Division

Integer tiles can be a simple and effective alternative to teaching your students the "rules" for operations involving integers. Most times, after sufficient practice, students will discover the patterns within adding, subtracting, multiplying and dividing integers, using integer tiles, on their own. It may not even be necessary for the teacher to identify the "rules" for them. In addition, they will comprehend what the meaning behind the "rules" is and understand why they work, making it easier for them to remember them when they are working independently.

Materials:

Integer tiles can be created easily and inexpensively. Here are a few suggestions.

- Tile spacers- Tile spacers can be purchased at the hardware store and already look like plus symbols. Simply divide the bag in half, keep one half as the positive tiles and make negative tiles out of the other half by slicing off two opposite ends.

- Counters- You could use different color counters (e.g. yellow and red) and label all the counters of one color with plus symbols and the other color with negative symbols

- Cardstock- Cardstock could be cut into small squares and labeled with either plus or minus symbols. You could make the tiles more easily identifiable by using different color card stock for the positive and negative tiles.

The integer tiles could then be divided into snack size baggies, as a class set, and used by your students during class as manipulatives. Students should also be encouraged to draw the tiles onto their classwork or homework as a way of demonstrating their work. Eventually, students won't rely on the tiles at all; they will become more efficient and either draw tiles on their paper or have discovered the patterns and not need them at all.

Representing integers with integer tiles:

1. Introduce each tile as representing one positive or one negative.

2. Ask students how they would represent different integers using the tiles.

   Examples: +3 = + + + -2 = − −

3. Ask students how they would represent zero using the tiles (note: no tiles would not meet the directions).

   Examples: 0 = ± or ± ± ± or ± ± ± ± ± ± ± ±

   *Each positive and negative could be referred to as a zero-pair.

   *They could use an infinite number of zero-pairs and still have zero.

4. Ask students how they could represent a variety of numbers using more than the minimum number of tiles.

   Examples:

   a. Represent (+3) with at least 7 tiles:

      + + + ± ± or + + + ± ± ± ± ± ± ± ±

   b. Represent (–2) with at least 4 tiles:

      − − ± or − − ± ± ± ±

   c. Represent (+5) with at least 6 tiles:

      + + + + + ± or + + + + + ± ± ± ± ±

   d. Represent (0) with at least 11 tiles:

      ± ± ± ± ± ± or ± ± ± ± ± ± ± ±

Multiplying Integers:

Remember that multiplication represents repeated addition. In elementary school students learn that 3 • 5 represents 3 groups of 5, then they repeatedly add what's in the groups. This is why the word "of" is many times identified as representing multiplication.

1. When multiplying integers:
   - The sign of the first value tells us if we're adding or taking away groups.
   - The sign of the second value tells us what tiles the groups consist of.

2. If you have to "take away" groups of tiles you don't have, you could add as many zero-pairs as needed until you have enough tiles.

Examples:

a. (+3) • (+4) reads, "Add 3 groups of 4 positive tiles".

(+ + + +) (+ + + +) (+ + + +)

Therefore, (+3) • (+4) = +12

b. (-3) • (+4) reads, "Take away 3 groups of 4 positive tiles".

Add at least 12 zero-pairs: + + + + + + + + + + + +
– – – – – – – – – – – –

(+ + + +) (+ + + +) (+ + + +)
(– – – –) (– – – –) (– – – –)

Therefore, (-3) • (+4) = -12

c. (+3) • (-4) reads, "Add 3 groups of 4 negative tiles".

(- - - -) (- - - -) (- - - -)

Therefore, (+3) • (-4) = -12

d. (-3) • (-4) reads, "Take away 3 groups of 4 negative tiles".

Add at least 12 zero-pairs: + + + + + + + + + + + +
– – – – – – – – – – – –

(+ + + +) (+ + + +) (+ + + +)
(– – – –) (– – – –) (– – – –)

Therefore, (-3) • (-4) = +12

Dividing Integers:

After exploring multiplication using integer tiles, dividing integers could be explained as the inverse of multiplying integers.

For example: If 3 • 5 = 15 then the inverse would be 15 ÷ 5 = 3 and vice-versa.

Therefore, any division problem could be solved if it's represented as the inverse of a multiplication problem.

Examples:

a. (+6) ÷ (+3) = (?) rewrite as: (?) • (+3) = (+6)

How many groups of 3 positive tiles would you have to add/take away to get 6 positive tiles?

(+ + +) (+ + +)

You would have to add 2 groups of 3 positive tiles. ? = +2

Therefore, (+6) ÷ (+3) = (+2)

b. (+6) ÷ (-3) = (?) rewrite as: (?) • (-3) = (+6)

How many groups of 3 negative tiles would you have to add/take away to get 6 positive tiles?

(+ + +) (+ + +)
(− − −) (− − −)

You would have to take away 2 groups of 3 negative tiles. ? = -2

Therefore, (+6) ÷ (-3) = (-2)

c. (-6) ÷ (+3) = (?) rewrite as: (?) • (+3) = (-6)

How many groups of 3 positive tiles would you have to add/take away to get 6 negative tiles?

(+ + +) (+ + +)
(− − −) (− − −)

You would have to take away 2 groups of 3 positive tiles. ? = -2

Therefore, (-6) ÷ (+3) = (-2)

d. (-6) ÷ (-3) = (?) rewrite as: (?) • (-3) = (-6)

How many groups of 3 negative tiles would you have to add/take away to get 6 negative tiles?

(- - -) (- - -)

You would have to add 2 groups of 3 negative tiles. ? = +2

Therefore, (-6) ÷ (-3) = (+2)

Multiplication Patterns:

Question your students on what kind of patterns they notice.

- When adding groups of positive tiles, is the answer positive or negative?
    - Is this always true?
    - Therefore, (+) • (+) = (+)
- When taking away groups of positive tiles, is the answer positive or negative?
    - Is this always true?
    - Therefore, (-) • (+) = (-)
- When adding groups of negative tiles, is the answer positive or negative?
    - Is this always true?
    - Therefore, (+) • (-) = (-)
- When taking away groups of negative tiles, is the answer positive or negative?
    - Is this always true?
    - Therefore, (-) • (-) = (+)

Division Patterns:

Develop division patterns by using the inverse of the multiplication patterns.

- If (+) • (+) = (+), then the inverse is (+) ÷ (+) = (+)
- If (+) • (-) = (-), then the inverse is (-) ÷ (-) = (+)
- If (-) • (+) = (-), then the inverse is (-) ÷ (+) = (-)
- If (-) • (-) = (+), then the inverse is (+) ÷ (-) = (-)